p^2*Charging*: Proactive Partial Charging for Electric Taxi Systems

Yukun Yuan, Desheng Zhang[†], Fei Miao[§], Jiming Chen[*], Tian He[‡], and Shan Lin
Stony Brook University, [†]Rutgers University, [§]University of Connecticut, [*]Zhejiang University, [‡]University of Minnesota

***Abstract*—Electric taxis (e-taxis) have been increasingly deployed in metropolitan cities due to low operating cost and reduced emissions. Compared to conventional taxis, e-taxis require frequent recharging and each charge takes half an hour to several hours, which may result in unpredictable number of working taxis on the street. In current systems, E-taxi drivers usually charge their vehicles when the battery level is below a certain threshold, and then make a full charge. Although this charging strategy directly decreases the number of charges and the time to visit charging stations, our study reveals that it also significantly reduces the availability of number of taxis during busy hours with our data driven analysis. To meet dynamic passenger demand, we propose a new charging strategy: proactive partial charging (p^2Charging), which allows an e-taxi to get partially charged before its remaining battery level is running too low. Based on this strategy, we propose a charging scheduling framework for e-taxis to meet dynamic passenger demand in spatial-temporal dimensions as much as possible while minimizing idle time to travel to charging stations and waiting time at charging stations. This work implements and evaluate our solution with large datasets that consist of (i) 7,228 regular internal combustion engine taxis and 726 e-taxis, (ii) an automatic taxi payment transaction collection system with total 62,100 records per day, (iii) charging station system, including 37 working charging stations over the city. The evaluation results show that p^2Charging improves the ratio of unserved passengers by up to 83.2% on average and increases e-taxi utilization by up to 34.6% compared with ground truth and existing charging strategies.**

I. Introduction

Electric taxis have been deployed in large scale in many cities for public transit with local governments' support and incentives, *e.g.*, Chicago [1], New York City [2] and Los Angeles [3]. For example, in Shenzhen, a city in China, an e-taxi fleet has passed the business breakeven point since 2013 [4]. Compared to conventional internal combustion engine taxis which have an average of around 300 miles on a full tank of gas, e-taxis travel between 60 and 200 miles on a full charge [5]. So, e-taxis require more frequent recharges. Our study with real e-taxi data traces shows that an e-taxi recharges more than three times per day on average. Also, different from conventional taxis which only need several minutes to fill their tanks, a full charge of an e-taxi takes as little as 30 minutes or up to several hours. Moreover, e-taxis usually have to wait for an available charging point at a charging station, given the limited number of charging points and stations. As a result, each e-taxi spends a significant amount of idle time at the charging stations. In fact, [6] shows that 48.75% of e-taxi drivers spend more than 3 hours at charging stations per day.

Such long idle time at charging stations reduces the availability of e-taxi service, resulting in unbalanced taxi supply and passenger demand. Especially, this happens during busy hours when passengers are waiting on the streets, but e-taxis are getting charged or waiting to be charged. Therefore, the timing and duration of each charge of e-taxis are critical to the quality of the e-taxi service. Our analysis reveals that most e-taxi drivers charge their vehicles only when their batteries are low, and more than half of taxi drivers charge their batteries to full on each charge. Although conducting full charge reactively can reduce the total number of charges, it also misses opportunities to serve more passengers, since during busy hours an e-taxi can find a passenger quickly if it stops charging when its battery is charged sufficiently high but not necessarily full.

To satisfy dynamic passenger demand, we propose a new charging strategy: proactive partial charging. Proactive partial charging suggests that an e-taxi can get partially charged rather than fully charged and get charged before its remaining energy is running too low. This strategy allows much more temporal flexibility for scheduling charging tasks so that we can allocate the taxi supply to match passenger demand and also reduce waiting time at charging stations. In this paper, we formulate the Electric Taxis Proactive Partial Charging Scheduling Problem and propose a proactive partial charging (p^2Charging) framework to schedule and coordinate when and where to charge, and how much energy to charge for each e-taxi. The objectives of our formulation include maximizing satisfied passenger demand and minimizing the cost of charging that includes the total driving time to charging stations and waiting time at charging stations for all e-taxis. Our solution utilizes predicted passenger demand and estimated waiting time at charging stations to find the global optimal charging decisions.

There are a number of research works on electric vehicles and scheduling algorithms [7], [8], [9], [10], [11], [12]. These works provide valuable insights into EV charging problems. For example, Dong et al. [13] provides a scheduling algorithm to achieve bounded waiting time in the charging station. However, it adopts the reactive charging strategy, which schedules an e-taxi when its battery is below a fixed level; some other works [8], [9], [7], [14], [15], [16] employ the full charging strategy, which assumes every charge is a full charge. Compared to existing charging solutions, including reactive full charging [13], reactive partial charging [10], and proactive full charging [15] strategies, our solution can better balance

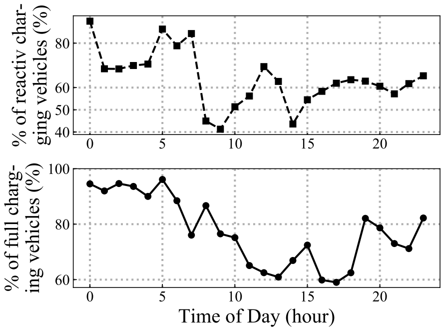

Fig. 1: Charging behaviors analysis

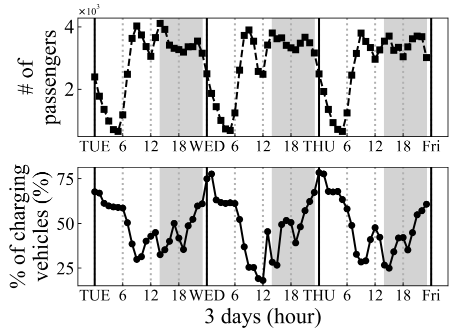

Fig. 2: Mismatch between Passenger Demand and E-Taxi Supply

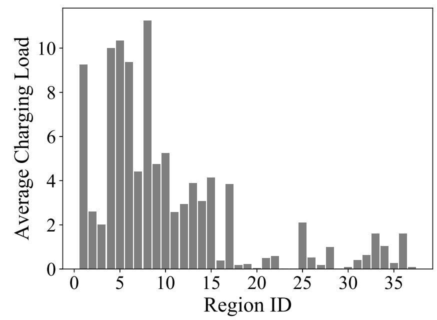

Fig. 3: Charging demand distribution

taxi supply with passenger demand with little overhead.

To the best of our knowledge, our work is the first to schedule e-taxis charges with a proactive partial charging strategy. Compared to previous charging solutions that use fixed thresholds to decide the timing and duration for each charge, actually proactive partial charging is a more generic type of charging strategy, which can be reduced to reactive and full charging with special parameter settings.

One issue for conducting partial charge is the increased number of charges, but actually, the rapid advancements of battery technology allow for more and more recharges [17], [18], [19]. Moreover, recent research [20], [21] shows that deep discharge and high charge current actually shorten lithium battery life; on the contrary, taking a discharge rate consistently to 50% can improve the battery life expectancy to 3 or 4 times compared with 100% discharge.

The contributions of this paper are listed as follows:

- To the best of our knowledge, we are the first to propose the proactive partial strategy for charging e-taxis. Proactive partial charging allows full charging tasks to be divided into small partial charging tasks, enabling more flexible charging schedules and better taxi services with little overhead.
- We formulate the Electric Taxis Proactive Partial Charging Scheduling Problem, and model variable battery levels in our formulation. Different from previous taxi charging works that only consider binary battery levels (empty and full), our variable battery level modeling provides a new dimension for scheduling algorithm design.
- The charging process of e-taxis is tightly coupled with the dispatch process of taxi service. To integrate our charging solution with existing taxi dispatch systems, we propose the p^2Charge framework that employs a receding horizon optimization approach to coordinate both processes simultaneously with real-time multi-source data.
- Our study is based on one of the most comprehensive datasets for taxis that consist of (i) 7,228 regular internal combustion engine taxis and 726 e-taxis, (ii) an automatic taxi deal collection system with a total 62,100 records per day, (iii) charging station system, including 37 working charging stations over the city, Shenzhen, where our data is collected. Our data-driven evaluation shows that compared to ground truth and existing charging strategies, including reactive full charging [13], reactive partial charging, and proactive full charging [15] strategies, p^2Charging improves the ratio of unserved passengers by up to 83.2% on average and improve e-taxi utilization by up to 34.6% on average.

II. Data-driven Charging Strategy Analysis

In existing e-taxi systems, an e-taxi typically requires multiple recharges per day. E-taxi drivers choose when, where, and how long to charge the car battery based on their experiences. In this section, we reveal that most drivers practice the reactive full charging: get a full charge only when the remaining energy is low. Our data driven analysis shows that such an uncoordinated greedy charging strategy is inefficient, which results in mismatch between passenger demand and e-taxi supply especially during rush hours. The details of the datasets we used is shown in Section V-A.

We use 20 minutes time slots to partition the datasets. If one e-taxi driver charges his vehicle when its battery level is below 20% [22], then we consider it as reactive charging. While if a vehicle's battery level is above 80% after charging, it is considered to have a full charge. We calculate the battery level of each e-taxi by applying an energy consumption model [23]. Figure 1 plots the percentage of reactive and full charging vehicles of one day. We can see that on average 63.9% of drivers practice reactive charging and on average 77.5% of drivers practice full charging. We can also observe that some drivers' charging behaviors change during the day that. Most drivers conduct reactive and full charging in the morning and evening of a day, possibly due to low passenger demand; while during 10 am to 12 pm, the percentage of reactive charging vehicles increases and that of full charging vehicles decreases. This is because taxis nearly use up energy after trips in the morning, and drivers can only charge the battery during limited lunch time, which also confirms the results in research [4].

We also analyze the dynamics of passenger demand and the percentage of charging e-taxis over time. Figure 2 plots the number of passengers and the percentage of charging vehicles over three days, here we use the number of passengers who were picked up to represent the passenger demand. We have several observations of Figure 2. The first one is the daily passenger demand and vehicle charging patterns are similar

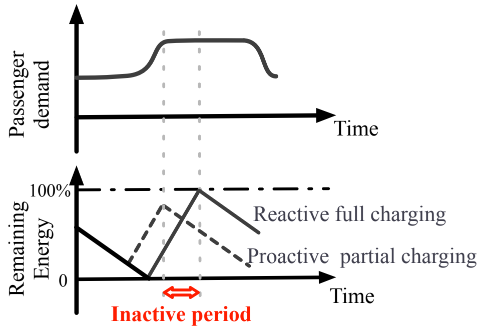

Fig. 4: Comparison of reactive full and proactive partial charging

over these days, most e-taxis get charged at night and get on the road during the day. The second observation is while the passenger demand is consistently high during the day, the percentage of charging vehicles varies significantly over the day. We can see clear mismatches between passenger demand and working taxi supply in the afternoon and evening hours, as highlighted in grey in the Fig. 2. This is because many e-taxis get complete discharged after the morning hours, then they have to get charged one by one at limited charging stations.

In addition, every one of 37 working charging stations is regarded as the center of one region and any location in the city belongs to the region with the nearest center. We analyze the geographical distribution of charging demand. Since the number of charging points varies in different regions, we use the ratio between total charging requests and the number of charging points in the region as a metric, called average charging load. Figure 3 plots the average charging load in all the regions. We can see charging load varies a lot among regions, *e.g.*, the average charging load of region 5 is nearly 5.1 times larger than that of region 25. This suggests that charging demand is unbalanced in different regions, it is also important to balance the charging demand across different regions to reduce e-taxis' waiting time at charging stations.

III. p^2Charging Overview

A. Proactive Partial Charging Strategy

To address the mismatch between passenger demand and e-taxi supply, it is essential to employ a new charging strategy: proactive partial charging. Proactive partial charging divides full charging tasks into small partial charging tasks and allows flexible charging schedules to adapt taxi supply to spatiotemporal dynamic passenger demand, therefore improving e-taxis fleet's service quality with little overhead.

Figure 4 shows an example to demonstrate the key idea of proactive partial based charging schedules. We can see that with the reactive full charging strategy (straight line), an e-taxi operates until its battery depletes and then goes to a charging station to get a full charge. But when the passenger demand starts rising during the rush hours, the taxi is receiving a full charge and stays inactive, missing opportunities to pick up more passengers while reducing taxi supply on the street. On the contrary, with the proactive partial charging strategy (dashed line), a taxi does not need to wait long periods until

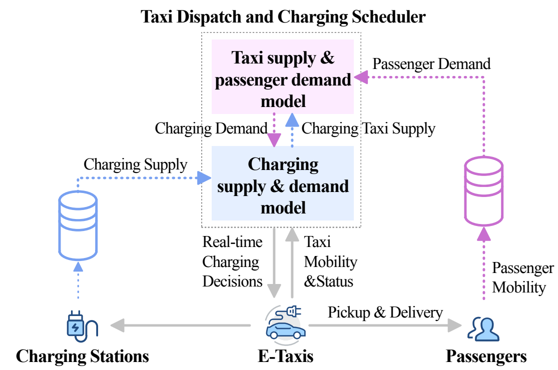

Fig. 5: p^2Charging framework

its battery depletes or is fully charged. To prepare for the rush hour, it can get charged for a period even before its battery is not depleted. So, it will have a sufficient amount of energy to operate through the rush hours to pick up more passengers. Plus, it can stop charging when passenger demand increases even when its battery is not fully charged. Compared to reactive full charging, the flexible timing and variable duration of charging allows us to make much more efficient charging schedules, to match taxi supply with passenger demand. We note that an individual e-taxi can be carefully scheduled to match passenger demand in rush hours even with reactive full strategy. However, when all e-taxis practice this strategy, they have to queue up at a charging station, which has a limited number of charging points, resulting in an even longer inactive period and reduced taxi supply due to the long waiting period.

B. p^2Charging Architecture

Based on the proactive partial charging strategy, we design a p^2Charging framework to schedule and coordinate the charging tasks of all e-taxis. Figure 5 shows an overview of the p^2Charging architecture. This architecture is designed based on the existing e-taxis systems of metropolitan cities, in which e-taxis are equipped with networked GPS, fare meter, and communication devices to upload real-time status, *e.g.*, current location and occupancy status to a dispatching center for monitoring and dispatching purposes [24], [25], [26], [27].

In p^2Charging, taxi scheduler periodically updates the status of current working e-taxis, *e.g.*, location, remaining energy and occupancy status, according to the uploaded e-taxis' status and then schedules when, where and how long to charge them to meet spatiotemporal passenger demand. E-taxis follow the charging decisions obtained from the taxi scheduler to charge their battery. The taxi scheduler uses both passenger demand and taxi supply model and charging supply/demand model to make scheduling decisions.

p^2Charging is driven by real-time multi-source data, which is provided by existing infrastructures, including e-taxi system and charging stations. These datasets contain rich spatiotemporal information about passenger mobility patterns and the demand and supply in either urban taxi serivce or charging system. To integrate this information into real-time scheduling,

p^2Charging employs a receding horizon control (RHC) framework to adapt charging decisions based on both current and future passenger demand and charging supply. Our framework allows taxi scheduler to specify multi-objective optimization goals under the charging system constraints and taxi system requirements. p^2Charging makes decisions for a group of e-taxis simultaneously by solving an optimization problem repeatedly at each iteration step of the RHC framework and then updates charging commands periodically. The objectives of p^2Charging are: (i) meeting dynamic passenger demand with supply in both spatial and temporal dimensions; (ii) minimizing the charging cost, *e.g.*, idle driving time to charging stations and waiting time at charging stations.

IV. Proactive Partial Charging Problem Formulation and Algorithm Design

In this section, we formulate the e-taxi proactive partial charging problem that decides charging schedules for each e-taxi. Informally, our goal is to meet dynamic passenger demand in spatial-temporal dimensions as much as possible and minimize cost related to scheduling e-taxis for charging. To address this problem, we design a receding horizon based scheduling algorithm that utilizes predicted passenger demand and waiting time at charging stations to find the global optimal charging schedules for all e-taxis.

Definition 1 (Proactive Partial Charging Scheduling Problem (P^2CSP))**.** *Given the spatiotemporal distribution of passenger demand, the location of charging stations and the number of charging points at each charging station, and initial energy status of all e-taxis in a city, how to decide when, where and how long each e-taxi should be charged for serving as many passenger as possible while minimizing the idle time to travel to charging stations and waiting time for free charging points at charging stations.*

The formal objectives, constraints and mathematical formulation will be introduced in the following subsections.

A. E-taxi network

We discretize time and space. Given current time slot t, we consider future m time slots for e-taxis charging scheduling, where one time slot is indexed by k, $(k = t, ..., t + m - 1)$. Suppose that the entire area of a city is partitioned into n regions according to some specific methods, such as administrative sub-districts [28], grid file [29] and quad-tree [30].

Each e-taxi has one of three states at any time slot: working, waiting, and charging, where working means the e-taxi is on the road to search or deliver passengers, waiting indicates the e-taxi is waiting for a free charging point at a charging station, and charging represents that the e-taxi connects to a charging point to charge its battery.

Here the remaining energy of an e-taxi is discretized into L levels. If an e-taxi works for one time slot, its remaining energy will decrease L_1 levels. L_2 represents the number of energy level increase if an e-taxi is charged for one time slot. The remaining energy does not change under waiting state.

Decision variables: In our problem, taxi scheduler needs to decide when, where and how long to charge the battery, which should be reflected on the decision variables. To represent when and where to charge, we define $X_{i,j}^k$ as the number of e-taxis dispatched from region i to j during time slot k for charging. The e-taxis with different initial energy level have a different range of charging duration. Given the initial energy level l, the maximum energy level L and the number of energy level increased when charged for one time slot L_2, the possible charging duration q is within $[1, \lfloor (L-l)/(L_2) \rfloor]$, meaning that if the initial energy level is larger than $L-L_2$, the taxi will not be charged for one time slot. Finally, we extend the decision variables from $X_{i,j}^k$ to $X_{i,j}^{l,k,q}$ to describe how many l-th energy level e-taxis are dispatched from region i to j during time slot k for charging q future time slots.

$X_{i,j}^{l,k,q}$ shows the consideration of proactive and partial charging simultaneously. The range of l, $[1, L]$, demonstrates that e-taxis with any energy level are considered for charging, showing proactive charging. Given the initial energy level l, all possible charging duration is considered, reflecting partial charging.

B. Passenger demand and taxi supply

The key objective of e-taxi service is to meet passenger demand with sufficient taxi supply. In e-taxi networks, taxi supply varies since e-taxis become out of service during its charging duration. In this subsection, we model passenger demand, taxi supply, and how charging schedules affect taxi supply.

Passenger demand: With historical dataset of taxi GPS and passenger transaction, we extract dynamic passenger demand information, such as passenger demand during rush or non-rush hours and in busy areas. We assume that during time slot k, the passenger demand that we want to serve by current available e-taxis at region i is denoted by r_i^k. These are the demands that we want to meet during time slot $t, ..., t+m-1$ with current available e-taxis.

Taxi supply: We study how taxi supply changes with charging decisions. $V_i^{l,k}, O_i^{l,k} \in \mathbb{R}_+$ are defined as the number of vacant and occupied l-th level e-taxis at region i at the beginning of time slot k before being dispatched for charging respectively. At each step of iteration, we first update the real-time sensing information, such as GPS locations, occupancy status, and energy status of all e-taxis, and $V_i^{l,t}$ and $O_i^{l,t}$ are provided by real-time data. Let $S_i^{l,k}$ be the total number of available e-taxis at the l-th level within region i during time slot k after scheduling.

$$
\begin{aligned}
S_i^{l,k} &= V_i^{l,k} - \sum_{j=1}^{n} \sum_{q=1}^{\lfloor (L-l)/L_2 \rfloor} X_{i,j}^{l,k,q}, \; k = t, ..., t+m-1 \\
V_i^{l,k+1} &= \sum_{j=1}^{n} Pv_{j,i}^k S_j^{l+L_1,k} + \sum_{j=1}^{n} Qv_{j,i}^k O_j^{l+L_1,k} + U_i^{l,k+1} \\
O_i^{l,k+1} &= \sum_{j=1}^{n} Po_{j,i}^k S_j^{l+L_1,k} + \sum_{j=1}^{n} Qo_{j,i}^k O_j^{l+L_1,k}
\end{aligned}
\tag{1}
$$

where $Pv_{j,i}^k, Po_{j,i}^k, Qv_{j,i}^k, Qo_{j,i}^k \in [0,1]$ are region transition matrices describing the taxis' mobility pattern between different regions during time slot k: $Pv_{j,i}^k(Po_{j,i}^k)$ describe the probability that one vacant taxi starting from region j at the beginning of time slot k will travel to region i and become vacant (occupied) at the beginning of time slot $k+1$. Similarly, $Qv_{j,i}^k(Qo_{j,i}^k)$ describe the probability that one occupied taxi starting from region j at the beginning of time slot k will travel to region i and become vacant (occupied) at the beginning of time slot $k+1$. Both current free and occupied e-taxis are considered for charging during future m time slots. The region transition matrices are learned from historical data by frequency theory of probability, and satisfy that:

$$\sum_{i=1}^{n} Pv_{j,i}^k + Po_{j,i}^k = 1, \ \sum_{i=1}^{n} Qv_{j,i}^k + Qo_{j,i}^k = 1$$

Note that previous work has developed multiple ways to learn passenger demand and taxi mobility patterns [31], [32], [33]. With perfect knowledge of passenger demand and taxi mobility patterns, we can set a large receding control horizon to control charging behaviors in a long time period. However, it is hard to have perfect predictions practically, since large accumulated prediction error over time may affect the performance negatively.

In Equation 1, $U_i^{l,k} \in \mathbb{R}_+$ is the number of e-taxis which finish charging at the beginning of time slot k in region i with energy level l. Here, we assume that once an e-taxi finishes charging in region i, it will be ready to pick up passenger at the beginning of next time slot. It is clear that $U_i^{l,k}$ is related to charging demand $X_{i,j}^{l,k,q}$ and charging supply p_i^k, and taxi supply is considered with charging demand/supply simultaneously. We will introduce how to calculate $U_i^{l,k}$ in the following charging demand/supply part.

C. Charging demand and supply model

With limited charging infrastructure distributed in the city, an e-taxi's waiting time is affected by the number of e-taxis in front of it at the charging station and their charging duration. In this subsection, we model the relation between charging demand and supply, and how to derive waiting time based on charging demand and schedules.

Charging supply: Although there are fixed number of charging points located in each region, the number of free charging points may vary from time slot t to $t+m-1$ due to existing waiting or charging e-taxis at charging stations. Let p_i^k denote the number of free charging points in region i during time slot k. At the beginning of time slot t, taxi scheduler updates the existing charging demand from current waiting or charging e-taxis in each region according to e-taxis' GPS trajectory and previous charging decisions. Then p_i^k is equal to total number of charging points minuses existing charging demand in region i during time slot k. We note that in existing infrastructure, charging stations are built over the city according to the same standard and each charging station may have different number of charging points.

Charging demand: With the assumption that all e-taxis follow charging decisions, charging demand consists of existing charging demand in each region and future charging demand decided by taxi scheduler at the beginning of time slot t, *i.e.*, $X_{i,j}^{l,k,q}$. The existing charging demand is considered when updating charging supply, and our problem decides the future charging demand with constrained charging supply p_i^k.

According to the charging decision $X_{i,j}^{l,k,q}$, we define $D_i^{l,k,q}$ as the number of l-th energy level e-taxis dispatched to region i during time slot k with q time slots charging duration, where

$$D_i^{l,k,q} = \sum_{j=1}^{n} X_{j,i}^{l,k,q} \tag{2}$$

Waiting time estimation: Due to a limited number of charging points in each region, e-taxis may need to wait for a free charging point. Here we consider all the charging points homogeneous, since currently local authorities built all charging points and e-taxis with the same standards [34], [35].

According to existing charging practices, if e-taxis are dispatched to the same region during different time slots, they are scheduled by first-come, first-serve. In the same time slot, they are scheduled by the shortest task first, meaning that the e-taxi with shorter charging duration is scheduled with higher priority. For simplicity, we assume that if one e-taxi is dispatched to one region at the beginning of a time slot, it will arrive within this time slot. Later we introduce one constraint to make sure that e-taxis will not be dispatched to a region that they cannot arrive within one time slot.

To represent the charging finish time of e-taxis, we define $Y_i^{l,k,q,k'}$ as the number of e-taxis that are dispatched to region i during time slot k and finish charging q time slots by the beginning of time slot k'. For l-th energy level e-taxis dispatched to region i at time slot k for charging q time slots, they may not finish charging by the end of optimization time horizon, and this number is denoted as $Du_i^{l,k,q} = D_i^{l,k,q} - \sum_{k'=k+q}^{t+m} Y_i^{l,k,q,k'}$. Then we constrain that $Du_i^{l,k,q} \geq 0$, meaning:

$$D_i^{l,k,q} - \sum_{k'=k+q}^{t+m} Y_i^{l,k,q,k'} \geq 0, \sum_{j=1}^{n} X_{j,i}^{l,k,q} - \sum_{k'=k+q}^{t+m} Y_i^{l,k,q,k'} \geq 0$$

For e-taxis that satisfying the definition of $Y_i^{l,k,q,k'}$, according to the scheduling discipline, some e-taxis should be charged before them, and we define the number of e-taxis with higher charging priority than them as $Db_i^{k,q}$:

$$Db_i^{k,q} = \sum_{k_1=t}^{k-1} \sum_{l=1}^{L} \sum_{q_1=1}^{\lfloor (L-l)/L_2 \rfloor} D_i^{l,k_1,q_1} + \sum_{l=1}^{L} \sum_{q_1=1}^{\tilde{q}} D_i^{l,k,q_1} \tag{3}$$

where $\tilde{q} = \min\{q-1, (L-l)/L_2\}$. The first part represents the number of e-taxis dispatched to region i before time slot k and the second part denotes the number of e-taxis dispatched to the same region i during the same time slot k with shorter charging duration.

If some e-taxis reach region i at time slot k with charging length q and finish charging at time slot k', there should be an amount of e-taxis which finish charging before time slot $k'-q$ and arrive at region i no later than time slot k. Such amount of e-taxis is denoted as $Df_i^{k,q,k'}$ and calculated by:

$$Df_i^{k,q,k'} = \sum_{l=1}^{L}\sum_{k_1=t}^{k-1}\sum_{q_1=1}^{\hat{q}}\sum_{k_1'=k_1+q_1}^{k'-q} Y_i^{l,k_1,q_1,k_1'} + \sum_{l=1}^{L}\sum_{q_1=1}^{\bar{q}}\sum_{k_1'=k+q_1}^{k'-q} Y_i^{l,k,q_1,k_1'} \quad (4)$$

where $\hat{q} = \min\{\lfloor (L-l)/L_2 \rfloor, k'-q-k_1\}$ and $\bar{q} = \min\{\lfloor (L-l)/L_2 \rfloor, k'-q-k, q-1\}$. In Equation 4, the first and second part represent the number of e-taxis that finish charging before $(k'-q)$ and are dispatched to region i before and at k respectively. In summary, for e-taxis satisfying $Y_i^{l,k,q,k'}$, at the beginning of time slot $(k'-q)$, the number of e-taxis that are still connected with one charging point is: $Db_i^{k,q} - Df_i^{k,q,k'}$. Considering the limited number of charging points, we have the following constraint:

$$Db_i^{k,q} - Df_i^{k,q,k'} + \sum_{l=1}^{L-q\times L_2} Y_i^{l,k,q,k'} \le p_i^{k'-q} \quad (5)$$

Therefore, we consider the taxi supply provided by charged e-taxis, $U_i^{l,k}$ in the previous passenger demand and taxi supply model:

$$U_i^{l,k} = \sum_{q=1}^{\lfloor (l-1)/L_2 \rfloor}\sum_{k_1=t}^{k-q} Y_i^{l-q\times L_2,k_1,q,k} \quad (6)$$

D. Problem formulation

According to the problem statement in Definition 1, we want to schedule e-taxis for charging with satisfying as many passengers as possible and minimizing the idle driving time and waiting time for a free charging points. The decision variables have already been studied previously and we formulate the objectives, constraints and mathematical description of our problem in the following part.

Objective: Satisfying the demand by allocating taxi supply across the network in spatial-temporal dimensions is one type of service quality metric in taxi dispatching system [36], autonomous mobility-on-demand system [25], [37] and subway-bus scheduling [38]. Whereas, because of the large passenger demand during peak hours, such as 8:00∼9:00 and 17:00∼19:00, the supply that taxi network can provide may not satisfy the passenger demand. In this work, we consider the number of unsatisfied passengers in each region during each time slot as the measurement of meeting demand with supply, denoted by $\max\{0, r_i^k - S_i^k\}$, where $S_i^k = \sum_{l=1}^{L} S_i^{l,k}$. The objective of meeting demand with supply in both spatial and temporal dimensions is formulated as:

$$J_s = \sum_{k=t}^{t+m-1}\sum_{i=1}^{n} \max\{0, r_i^k - \sum_{l=1}^{L} S_i^{l,k}\} \quad (7)$$

We aim to minimize this objective function.

Besides satisfying passenger demand, we also consider minimizing the cost of scheduling e-taxis for charging, including the idle driving time to charging stations and waiting time for a free charging point. Given the spatial structure of one city, we define $W_{i,j}^k \in \mathbb{R}$ as the weight matrix describing the driving time from region i to region j during time slot k, which can be estimated more precisely by incorporating historical and real-time data [39], [40]. Then the total idle driving time to charging stations is:

$$J_{idle} = \sum_{k=t}^{t+m-1}\sum_{l=1}^{L}\sum_{i,j=1}^{n}\sum_{q=1}^{\lfloor (L-l)/L_2 \rfloor} X_{i,j}^{l,k,q} W_{i,j}^k \quad (8)$$

For e-taxis satisfying $Y_i^{l,k,q,k'}$, their waiting time for one free charging point is $k'-q-k$. Meanwhile, for the e-taxis which do not finish charging by the end of time slot $t+m-1$ (the beginning of time slot $t+m$), we use $t+m-k-q+1$, the lower bound of waiting time of these e-taxis as their waiting time. In conclusion, the total waiting time is:

$$J_{wait} = \sum_{i,l,k,q,k'} Y_i^{l,k,q,k'} \times (k'-q-k) + \sum_{i,l,k,q} Du_i^{l,k,q} \times (t+m-k-q+1)$$

Constraints: The distance every e-taxi can travel during one bounded time slot is also bounded, due to limited speed and traffic conditions. We define one constraint parameter, $c_{i,j}^k \in \{0,1\}$, such that $c_{i,j}^k = 0$, if region j can be reached from region i within time slot k, otherwise, $c_{i,j}^k = 1$. Then the following constraint

$$X_{i,j}^{l,k,q} c_{i,j}^k = 0, \quad l = 1, ..., L \quad (9)$$

represents that if region j cannot be reached from region i during time slot k, the number of scheduled e-taxis for charging should be 0.

For the e-taxis, the operation sustainability is one major concern. E-taxis' batteries discharge while driving and they should have enough energy to be operated on the road. With the assumption that the charging behaviors of each e-taxi follow the decisions of our charging scheduler, our scheduling decisions should ensure that the low energy e-taxis are charged. The following constraint

$$S_i^{l,k} = 0, \quad l = 1, ..., L_1 \quad (10)$$

ensures that the low energy e-taxis $l \le L_1$ at each region during each time slot are not used to pick up passengers in case of using up energy during one time slot on the road.

We define one weight parameter β when summing up the two objectives: (i) serving as many passengers as possible and (ii) reducing the idle driving time to charging stations and waiting time for a free charging point. To summarize, we formulate the following problem based on the previous definitions of decision variables, constraints, and objectives:

$$\begin{aligned} \min_{X_{i,j}^{l,k,q}, Y_i^{l,k,q,k'}} \quad & J = J_s + \beta(J_{idle} + J_{wait}) \quad (11) \\ \text{s.t.} \quad & X_{i,j}^{l,k,q} c_{i,j}^k = 0, \\ & S_i^{l,k} = 0, \ (1) \sim (6) \end{aligned}$$

E-taxis partial proactive charging scheduling problem, Equation 11, is a mixed-integer linear programming problem (MILP) which can be solved by branch-and-bound [41] and

Algorithm 1: ***E-taxi charging algorithm with real-time information for taxi scheduler***

Input: Duration of one time slot: t_1 minutes; time horizon m time slots; parameter L, L_1, L_2, β

Output: Control decision: $X_i^{l,t,q}, i \in [i, n], l \in [1, L], t \in [0, 24 * 60/t_1], q \in [1, \lfloor (L-l)/L_2 \rfloor]$

1: **while** At the beginning of each t_1-minutes time slot **do**

2: Update current time slot as t, sensor information for initial positions and energy status of vacant e-taxis $V_i^{l,t}$ and occupied e-taxis $O_i^{l,t}$;
Update the charging supply p_i^k, driving time matrix W^k and driving distance constraint parameters , $c_{i,j}^k$;
Update the passenger demand of every region to region pair based on historical data and real-time sensor information.

3: Solve the charging scheduling problem, Equation 11 to get the charging scheduling decision.

4: Send current time slot's charging decisions: $X_i^{l,t,q}$.

5: **end while**

6: **return** Charging decision

cutting-plan [42] algorithms. In our evaluation experiment, the global optimal solution can be obtained within 2 minutes using one multi-core PC by an existing solver, Gurobi [43].

E. Charging Scheduling Algorithm

Passenger demand and taxi mobility pattern can be learned from historical data, but they are not sufficient to calculate a charging scheduling solution due to dynamic positions of e-taxis and uncertainty of e-taxis' remaining energy. Hence, we design one receding horizon control (RHC) framework to adjust charging scheduling solutions and incorporate historical model with real-time sensing information.

The pseudo-code of RHC algorithm is shown in Alg. 1. Since we only calculate the number of a group of e-taxis, we assume that e-taxis with the same parameter, *i.e.*, region i, energy level l, are identical and randomly select one of them for charging based on the charging decisions. We update remaining energy of each e-taxi based on one energy consumption model [23] due to lacking such information in our dataset. However, remaining energy has already been displayed on the dashboard and e-taxis have communication devices. We argue that it is easy for an e-taxi company to collect real-time remaining energy information in the future. To update charging supply, we first infer the current charging demand based on the charging duration of current charging e-taxis and charging supply is equal to total number of charging points in each region minuses the charging demand.

We note that receding horizon control [44] has been used as a mathematical framework in some of the recent works [25], [37], [45] to adapt control decisions with real-time information. Although we use receding horizon control, the decision variables, objectives, and constraints are different from previous research, as they are defined by the specific charging scheduling problem that we study. Moreover, our problem considers multiple energy levels and charging duration for each e-taxi to conduct proactive partial charging which is totally different from previous work [36], [30] that only dispatching taxis to different regions for picking up passengers.

V. Evaluation

A. Data Description

The datasets we used consists of three parts as follows. **Existing charging station data**: the geographical distribution of existing charging stations is shown in [13]. Within the city, there are a total of 37 charging stations deployed and in use, and there is a different number of charging points at each charging station. We know the GPS location and number of charging points of each charging station.

Taxis' trajectory data: every taxi, including e-taxis and conventional taxis, has networked GPS device that can upload real-time location information every 30 seconds. One record in this dataset contains a plate number, a time stamp in seconds, GPS coordinates and an occupancy status. Based on this dataset and charging station information, we can infer when one e-taxi arrives at and leaves which charging station, and then all e-taxis' charging behaviors are mined.

Passengers' transaction dataset: it contains the information of each trip, such as when one passenger is picked up and dropped off, and the plate number of the taxi. By combining taxis' trajectory and passengers' transaction data, we can estimate the passenger demand in each region over the city during the different time slot of one day.

B. Methodology

To evaluate p^2Charging in a real-world scenario, we use the dataset described previously to conduct a trace-driven analysis. We partition the city into regions based on the location of charging stations, *i.e.*, each charging station is the center of one region and each location belongs to the region with the nearest center. From the dataset, we extract the origin/destination information of each trip, and then get the passenger mobility information between two regions in each time slot.

Since the dataset contains the GPS trajectory and pick-up and drop-off information of both regular and e-taxis, we use the number of passengers each regular taxi picks up to estimate the passenger demand of e-taxis for any two regions pair in each time slot. Due to lacking direct information of remaining energy of each e-taxi, we infer such remaining energy information by adopting an energy consumption model [23].

To show the effectiveness of p^2Charging, we compare it with the following existing solutions: (i) Ground: the ground truth extracted from the dataset; (ii) REC [13]: one reactive full charging solution whose charging threshold is 15% and one e-taxi is scheduled to the charging station with the minimum waiting time; (iii) proactive full charging [15]: given a group of e-taxis and charging stations, it always selects the e-taxi and charging station pair with the minimum idle driving time and waiting time; (iv) reactive partial charging: since [10] considers electricity price to adjust charging scheduling which is not considered in our problem, we reduce our p^2Charging with fixed charging threshold (20%) to this category.

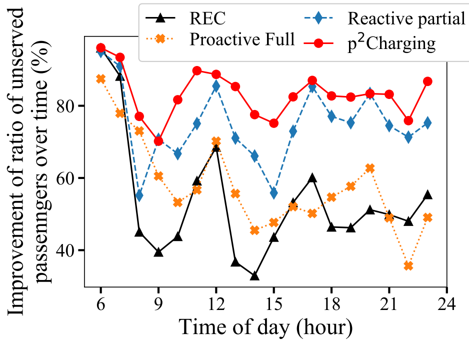

Fig. 6: Performance improvment over ground truth

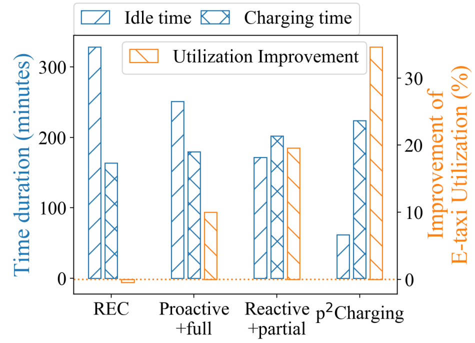

Fig. 7: Idle and waiting time, and e-taxi utilization

The performance metrics include: (i) ratio of unserved passengers: the number of unserved passengers over the total number of passenger demand; (ii) idle time: the sum of the idle driving time and the waiting time for each e-taxi; (iii) E-taxi utilization: 1-(idle time+total charging time)/total working time; (iv) improvement of ratio of unserved passengers: the performance improvement when comparing the ratio of unserved passengers by any one of four solutions and that in ground truth.

C. Results

In the experiment, the length of each time slot is 20 minutes and then the time horizon is 6 time slots. We assume that the driving time after one full charge is fixed (300 minutes) and set the parameters as $\beta = 0.1$, $L = 15$, $L_1 = 1$ and $L_2 = 3$.

1) Comparison of solutions: Figure 6 plots the performance improvement of ratio of unserved passengers over time. The average improvement of REC, proactive full, reactive partial and p^2Charging is 53.6%, 56.8%, 74.8% and 83.2%, respectively. If all taxis are e-taxis and drivers follow the charging scheduling of p^2Charging in the city where our data was collected, nearly 45,000 more passengers will be served per day based on the total passenger demand described in [46].

We also have several observations. The first one is partial charging provides the opportunity for more e-taxis to prepare well for the upcoming high passenger demand duration. A large number of e-taxis go to charge the battery after the operation in the morning from 12:00, and p^2Charging and reactive partial charging outperforms the other two solutions during high passenger demand period, 13:00$\sim$15:00. The reason is due to partial charging, the first arriving charging e-taxis end charging before 13:00, which also reduces the waiting time of waiting e-taxis to get enough energy as early as possible. The second observation is by proactive charging, e-taxis can charge during low passenger demand period to be ready for the following rush hours. All four solutions have similar performance during 7:00$\sim$8:00, and proactive charging allows some e-taxis to charge the battery during such low passenger demand period, and then offer more supply during rush hour, after 9:00. The last observation is considering the charging decisions of all e-taxis rather than conducting local optimal decisions can coordinate the charging behaviors of all e-taxis to achieve better global performance.

Figure 7 plots the idle time length, charging time length and improvement of e-taxi utilization compared with ground truth. p^2Charging reduces the idle driving time and waiting time by 81.2%, 75.4% and 64.1% compared with the other three solutions, respectively. We can conclude that: (i) partial charging reduces the waiting time of e-taxis by ending the charging process as early as possible; (ii) proactive charging decreases the number of waiting e-taxis at charging stations during high charging demand period. At the beginning of one day, most e-taxis are close to full energy after charging during the first hours of one day, and they may reach the charging threshold simultaneously during daytime by reactive charging. Compared with ground truth, the four solutions achieve a performance improvement of -0.4%, 10.0%, 19.6% and 34.6% respectively, meaning that by p^2Charging, an e-taxi has 135.4 minutes more on the road to serve passengers compared with the ground truth if one driver works 12 hours per day.

2) Remaining energy before and after charging: Figure 8 and 9 plot the CDF of remaining energy before and after charging respectively. Reactive full/partial charging and proactive/reactive full charging are not shown in two figures, since they use one fixed threshold to start or end charging, which will be a curve jump from 0 to 1 at a specified threshold. For ground truth, 80% e-taxis' remaining energy before charging is no more than 0.28, whereas, that of p^2Charging is 0.43. By p^2Charging, 40% e-taxis' remaining energy after charging is no more than 0.58 and that of ground truth is 0.8. It is concluded that compared with ground truth, p^2Charging achieves higher remaining energy before charging and lower energy after charging by proactive partial charging.

3) Overhead of p^2Charging: The overhead of p^2Charging is measured by number of charges. Figure 10 shows the number of charges of ground truth and by four solutions. We can see that one e-taxi needs to be charged nearly 9.7 times on average by p^2Charging, which is 2.78 times compared with that in ground truth. Considering the total energy needed to be charged for one e-taxi each day does not fluctuate between different charging strategies, both p^2Charging and reactive partial charging introduce a greater number of charges due to partial charging, while they introduce less idle time and higher e-taxi utilization as shown in Figure 7.

4) Impact of β: In Fig. 11 and 12, we show the impact of parameter β on the amount of picked-up passengers and the

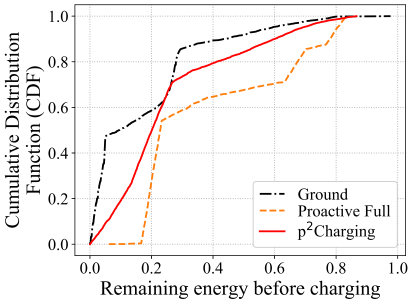

Fig. 8: **CDF of remaining energy before charging**

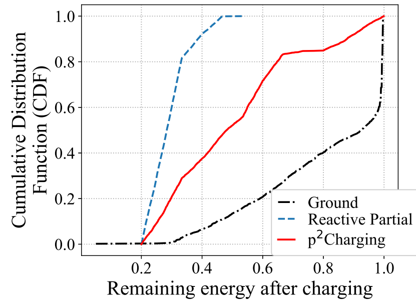

Fig. 9: CDF of remaining energy after charging

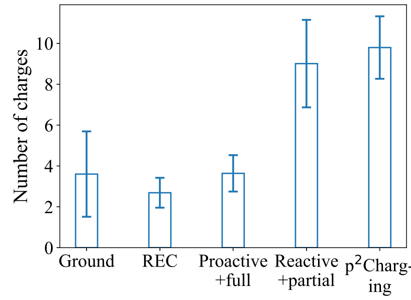

Fig. 10: Overhead of p^2Charging

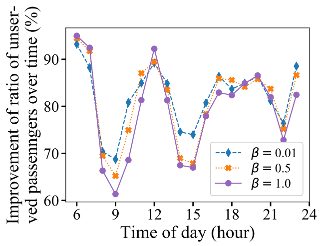

Fig. 11: Impact of β on ratio of unserved passengers

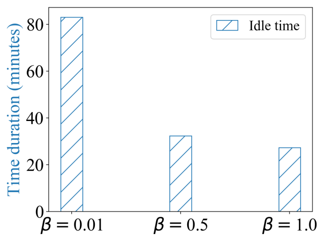

Fig. 12: Impact of β on idle time

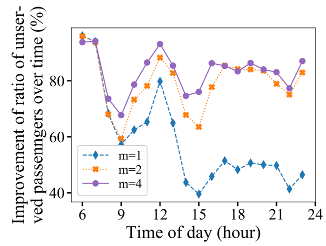

Fig. 13: Impact of time horizon

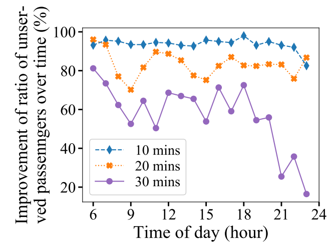

Fig. 14: Impact of update period

idle time for charging, including idle driving and waiting time. We set the β as 0.01, 0.5 and 1.0, the time slot as 20 minutes and the time horizon as 6 time slots. The observation is that the performance improvement of $\beta = 0.01$ outperforms that of $\beta = 0.5$ and 1.0 with average improvement by 4.3% and 13.8% respectively over the day. With the increase of β, the average idle time decreases, *e.g.*, $\beta = 1.0$ reduces the average idle time by 16.6% and 67.6% compared with $\beta = 0.5$ and 0.01. It is observed that there is a trade-off between serving more passengers and reducing idle time duration. To minimize the idle time duration, *i.e.*, increasing β, e-taxis are scheduled to charging station deployed in the suburban area, where the idle waiting time decreases a lot, but few passengers are served due to low passenger demand in such areas.

One important observation is that the performance improvement of $\beta = 0.01$ is worse than that of $\beta = 0.5$ and 1.0 during 6:00∼8:00 and 12:00∼13:00. The reason behind this is there exists high passenger demand during 8:00∼11:00 and 14:00∼16:00 and p^2Charging focuses on satisfying more passengers during high passenger demand time periods which sacrifices the performance before such periods with a small β.

5) Time horizon: Figure 13 plots the performance improvement of p^2Charging with a different prediction time horizon: 1, 2 and 4 time slots (20, 40 and 80 minutes). The observation is that the performance improvement of 4 time slots horizon outperforms that of 1 and 2 time slots horizon with average 24.5% and 4.1% more performance improvement respectively over the day. The reason for this observation is that a shorter time horizon means that only passenger demand and vehicles' energy status in the very recent future is considered, which misses opportunities to achieve better control. Specifically, long time horizon provides the opportunity to prepare the upcoming rush hours, 8:00∼10:00 and 14:00∼17:00, proactively.

6) Control update period: Figure 14 plots the performance improvement of p^2Charging with different update periods: 10, 20 and 30 minutes. The prediction time horizon is set to be 120 minutes. We can see that shorter update periods can increase the performance of p^2Charging, as it allows more frequent control decisions for passenger demand, and e-taxis' dynamic energy status and location changes: when update period length is 10 minutes, it achieves 10.3% and 36.3% more improvement on average compared with 20 and 30 minutes.

7) Evaluation Discussion: Due to charging e-taxis partially, it may exist that some e-taxis have no enough energy to bring passengers from origin to destination and then get stuck somewhere middle of the path. In the simulation, given the pickup time slot and region, we observe that there are at least 98.0% of e-taxis that can serve all passenger trips.

We assume that all e-taxis have the same battery capacity, charging speed and energy consumption model, which is supported by our data that e-taxis are the same car model in the city where our data was collected, and previous work also makes the same assumptions [13], [23], [11]. We can extend our problem formulation with different battery, charging and energy consumption models to describe each e-taxi.

In our dataset, the number of available e-taxis varies with time, *i.e.*, new e-taxis joining or leaving the system based on their working schedules. If such scenario exists during one time slot, our system can handle it by updating the number of available e-taxis and recomputing scheduling decisions for current available e-taxis at the beginning of the next time slot.

We use trajectory to infer the energy consumption of e-taxis. When one e-taxi is at one charging station, its status, waiting or being charged is estimated by queueing model described in the previous waiting time estimation part of section IV-C.

In the evaluation, we estimate the passenger demand for e-taxis based on the passengers that served by both regular and electric taxis in each time slot. We note that our system performance is affected by the ratio between number of e-taxis and number of charging points. The benefits of p^2charging will increase if the ratio decreases.

VI. Discussion

Implementation of p^2Charging: We focus on the technical approach for e-taxis dispatching, instead of providing incentives for drivers to participate in our dispatching effort. In practice, based on our interactions with Shenzhen transportation committee (which oversees all taxi companies and controls taxi medallion), we believe most of the drivers will participate this effort since all drivers are currently under the dispatching platform to pick up passengers using smartphones to make taxi reservations. Since our goal is to reduce the total charging time for all taxis, the drivers have the obligation for their taxi companies to follow their dispatching. If most drivers do not follow our dispatching, we can utilize the concept of virtual electricity inspired [47] for incentivzing them.

Battery lifetime: Battery lifetime is one concern of e-taxis' drivers. We adapt proactive partial charging which increases charging times but will not shorten the lifetime of battery. Based on [20], [21], deep discharges shorten lithium battery life and taking a discharge rate consistently to 50% can improve the battery life expectancy to 3 to 4 times compared with 100% discharge. [48] shows that partial charging is better than full charging and deep discharge wears the battery down.

Lesson learned: Based on our results, we learned a few valuable lessons: (i) partial charging can reduce the waiting time and offer more ready e-taxis for rush hours; (ii) proactive charging takes the opportunity to charge some extra e-taxis during non-rush hours to prepare for rush hours; (iii) coordination of e-taxis charging scheduling can improve the system efficiency by considering global optimal rather than local optimal solution one by one.

Potential impact: A charging scheduling coordination system is beneficial for promoting e-taxis service quality. With the development of autonomous vehicles, e-taxi companies will operate and dispatch a group of autonomous e-taxis around a city to deliver passengers. Hence, our charging scheduling system is valuable to improve the profit of e-taxi companies by reducing the impact of charging on serving passengers.

Future work: One of the future works will be incorporating passenger capacity of each vehicle and ride-sharing scenarios. The other direction is to consider shared charging infrastructure among different types of electric vehicles.

VII. Related Work

There are many works on electric vehicle charging, most of these works use fixed parameters such as battery levels to decide when to start and finish a charge.

In many other works [7], [14], [15], [16], every charge is considered as a full charge. [7] designs a real-time charging station recommendation system for e-taxis by large-scale GPS data mining, where one vehicle is scheduled if it sends a request no matter the remaining energy. [14] schedules charging activities spatially and temporally to minimize charging waiting time, where one vehicle is scheduled if minimal waiting time is achieved. [15] proposes electric vehicles charging scheduling algorithms to reduce the total charging time, in which vehicles with distinct remaining energy. [16] investigates the operations of an e-taxi fleet that accommodates only those trips for which advance reservations are made and decides the changeable remaining battery time on arrival at one charging station. These works provide valuable insights to the electric charging problem but having the full charge assumption missing opportunities to serve more passengers when vehicles are sufficient high but not fully charged, which is represented by our approach.

	Reactive	Proactive
Partial	[10]	p^2Charging (Our work)
Full	[7], [8], [9], [13]	[14], [15], [16]

TABLE I: Electric taxis charging strategy comparison

There are several papers allows a vehicle to be charged opportunistically. [10] considers the time-varying electricity price and electric taxis' future charging behaviors and then proposes one charging scheduler to minimize the charging cost of electric taxis. Each taxi is charged only when electricity price is below a given threshold and repeats deciding whether charging the battery every time unit. [11] and [49] consider the wireless power transmission technology that allows electric vehicles to be charged going through road segments where charging devices are installed. A route planner system is designed to enable in-motion charging for electric vehicles.

In summary, we classify the related work into four different classes as shown in Table I. Our work is the only one that proposes a novel proactive and partial charging scheduling for e-taxis that enable flexible charge schedules and provide better service quality for taxi passengers. Compared to previous charging solutions that use fixed thresholds to decide the timing and duration for each charge, proactive partial charging is a more generic type of charge strategy, which can be reduced to reactive and full charging with special parameter settings.

VIII. Conclusion

We investigate charging behaviors for e-taxi fleets with real-world datasets and identified that most e-taxis conduct reactive full charges, which misses opportunities to serve more passengers during busy hours and leads to long idle time at charging stations. To address this problem, we design a novel proactive partial charging strategy and show that much more efficient charging schedules can be realized with centralized dispatch. So, we design, implement and evaluate the p^2Charging framework for e-taxi fleet to meet dynamic passenger demand with real-time multi-source data. Trace-driven simulation demonstrates our solution achieves up to 83.2% performance improvement of the ratio of unserved passengers and increases e-taxi utilization by up to 34.6% compared with ground truth and existing charging strategies.

Acknowledgment

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